Stringy Geometry and Topology of Orbifold

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1 Introduction

There is an emerging new subject of mathematics, which we call "Stringy Geometry and Topology of Orbifold". The motivation of stringy geometry and topology of orbifold comes from orbifold string theory discovered by physicists Dixon, Harvey, Vafa and Witten more than fifteen years ago. Orbifold has been around in mathematics since 50's. However, the classical theory of orbifold is basically an extension of theory of smooth manifold. The stringy geometry and topology is different. Its main purpose is to study stringy properties of orbifold, which is unique for orbifold. Orbifold string theory model is an popular model in string theory. There are more than two hundred papers on hep-th whose title include orbifold. During last two years, its mathematical counterpart is under going rapid development. It is fair to say that new phenomenon is being discovered every month! The growth of its foundation and connections to other areas of mathematics is explosive. On the other hand, it is still in its relatively early stage of development. I estimate that less than quarter of physics papers on this subject has been seriously looked at by mathematicians. Therefore, it is a particularly a good time to enter this subject now. Unlike other physics inspired theories such as gauge theory and quantum cohomology where entry costs are high, this subject is relatively easy to learn. I will cover basic of this subject aided by large number of examples. Here is the plan

(1) Basic on orbifold.

(2) Orbifold vector bundle and good map

(3) Degree shifting and orbifold cohomology group

(4) Poincare pairing and cup product

(5) Discrete torsion and Inner local system

(6) Twisted orbifold cohomology

(7) Projective representation

(8) Twisted orbifold K-theory

(9) Orbifold string theory conjectures

If the time is allowed, following topics will be discussed

(10) Orbifold quantum cohomology

(11) Orbifold Floer homology and counting the fixed points of hamiltonian symplectomorphism on orbifolds.

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